Piero della Francesca and painting as a science

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We live in a complicated world. Mere temporal proximity is no guarantee of causal connections between events. However, when the historian is dealing with very large complexes of social and intellectual developments, such as those gathered together under the titles 'Renaissance' and 'Scientific Revolution', there does seem to be a certain plausibility in the suggestion that the two sets of changes, being relatively close in time and in geographical as well as intellectual scope, may indeed have some connections one with another. The possible connection need not be that the former phenomenon is in some sense cited as a 'cause' of the latter, but it would seem reasonable to hope that we might, at least, see some of the same social and intellectual forces at work in both. This is of interest to the historian of science because the Renaissance has been the subject of intensive scholarly examination which has, over the years, established a framework within which it is possible to investigate relations between developments in social and economic structures and the development of literature and the visual arts. There has been much less study of the history of science in the period that is usually designated as that of the Renaissance in the arts, namely the fifteenth and sixteenth centuries. In fact, in histories of science the term 'Renaissance' is applied to the analogous phenomenon of the rediscovery of Ancient Greek scientific texts and consequently refers to a rather later period, starting in the sixteenth rather than the fifteenth century. Moreover, whereas to historians of the arts the term Renaissance carries heroic overtones, to historians of science the period of recovery of texts presents a more confused picture, seemingly not so much a period in its own right as a preliminary to the huge intellectual changes of the Scientific Revolution.

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What I propose to look at in the present study is a matter that does not involve the recovery of texts but rather a series of changes that affect mathematics and its relation to other intellectual disciplines. Such changes are a recognised component of the Scientific Revolution. For instance historians usually give some importance to Galileo Galilei's extension of the use of mathematics into areas formerly reserved for largely non-mathematical investigation by the natural philosopher, such as the movement of heavy bodies. Curiously enough, however, it is not quite so commonplace to make a connection between Galileo's much discussed use of experiment and his father's use of experiment in his discussion of the theory of music. A learned theoretician of music, a musicus, such as Vincenzo Galilei (c.1520 - 1591), was expected to have an acquaintance with musical practice. Music was one of the four mathematical sciences of the quadrivium, but it nevertheless had well established connections with the craft of the executant musician. There was thus nothing particularly surprising in Vincenzo's use of experimental evidence in support of his musical theory. However, the application of this craft practice by Galileo Galilei (1564 - 1642) in a different learned area was, to his contemporaries, distinctly suspect. It is, of course, partly because of the specialisation patterns of our own time that historians, trained within that system, tend to import it into the past. Thus historians of science do not necessarily study the history of music, which is, in any case, often written in a way that makes it highly opaque to people who do not have a present-day musical training. This importation of twentieth-century academic divisions has also had an effect upon studies of certain areas of the history of mathematics. For example, the history of perspective in the fifteenth and sixteenth centuries has largely been examined by historians of art, because it seemed important to them, whereas historians of the mathematics of this period have on the whole been chiefly interested in algebra, whose development had momentous effects upon the later evolution of mathematics.

In fact, the mathematics of the fifteenth century is not well covered in the learned literature. There is much darkness in between the narrow beams of light shed by specialised studies. The present study will be concerned with an area that shows connections with both earlier and later periods, namely the tradition of practical mathematics, the kind of mathematics that was learned by prospective merchants and prospective craftsmen. It was taught not only by private tutors (for the rich) but also in 'abacus schools' set up at public expense in various communities.[1] The connection with the medieval period is entirely obvious: most of the problems proposed to pupils in abacus schools are derived from an Islamic tradition that reached the Latin West through the work of Leonardo of Pisa (c.1170 - c.1240, sometimes called Fibonacci), whose Liber abaci gave its name to abacus schools while his Liber quadratorum is usually credited with having introduced the so-called 'Arabic' numerals whose use was characteristic of the abacus tradition. One connection of abacus mathematics with the mathematics of the later period is also obvious: the publication of the Ars magna (1545) of Girolamo Cardano (1501 - 1576) certainly marks its author's determination to place algebra in a learned Latin context, derived though it was from the vernacular tradition of textbooks of commercial arithmetic. However, commercial arithmetic (and the remainder of practical mathematics) certainly also continued as a separate tradition, fading out slowly from the mid seventeenth century onwards.[2]

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[1] We are best informed about what went on in the region of Tuscany, where there was, for example, a public abacus school in the city of Arezzo during most of the fifteenth century. See Robert Black, 'Humanism and Education in Renaissance Arezzo', I Tatti Studies (Essays in the Renaissance), 2, 1987, 171 - 237.

[2] The fading out of practical mathematics as a separate tradition is discussed in more detail in J. V. Field, The Invention of Infinity: Mathematics and Art in the Renaissance, Oxford: Oxford University Press (in press).

We have already noted Galileo's interest in the practical craft elements of music. He seems also to have been acquainted with practical mathematics, namely commercial arithmetic. Whether he was actually taught this kind of mathematics as a boy is open to conjecture, but in his Dialogo sopra i due massimi sistemi del mondo (Florence, 1632) he refers to commercial arithmetic for its assumption that the ducats in the real world will behave in exactly the same way as those in the school problem.[3] In context, this becomes a serious assertion, against some unnamed philosophers, of the applicability of mathematics to problems in the real world. Galileo is, of course, rightly famed for his skill in making telling points in a rhetorical fashion — and one may suspect that this verbal dexterity helped to make him enemies — but in the present instance it seems he has in fact merely given characteristically pithy expression to a notion that was of some importance in the learned world. For instance, there was wide acceptance of the usefulness of algebra, and its intellectual standing rose steadily throughout the sixteenth and seventeenth centuries, despite the fact that its philosophical foundations were decidedly shaky. A kind of mathematical common-sense seems to have imposed silence upon practitioners.[4]

Piero's education and writings

Piero della Francesca (c.1412 - 1492) was the eldest surviving (but probably the second born) son of a successful merchant whose family had been active citizens

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3 G. Galilei, Dialogo sopra i due massimi sistemi del mondo, Florence, 1632, p. 202; translated as Dialogue concerning the two chief world systems, tr. Stillman Drake, Berkeley (Cal.), 1953, pp. 207 - 8.

4 See J. V. Field, 'The relation between geometry and algebra: Cardano and Kepler on the regular heptagon', in E. Keßler (ed.), Girolamo Cardano: Philosoph, Naturforscher, Arzt, Wiesbaden, 1994, 219 - 242.

of the small town of Borgo San Sepolcro for several generations. Borgo had been ruled by Siena, but in the fifteenth century control passed to Florence, and the town eventually belonged to Tuscany (it is now called Sansepolcro). Piero's father, Benedetto, probably expected his son would join him in the family business, so it would have been entirely natural for the boy to have been given some instruction in abacus mathematics. This instruction would have been in the vernacular, that is in Tuscan. However, there is evidence that in later life Piero could also read and even write a little Latin.[5] He would not have learned Latin in an abacus school, so it seems possible that at least part of his education was by a private tutor. In any case, Benedetto was no doubt delighted to find that Piero showed an aptitude for mathematics.

The earliest biography of Piero is that given by Giorgio Vasari (1511 - 1574) in his collection of Lives of artists: Le Vite dei piu eccellenti architetti, pittori e scultori italiani da Cimabue a tempi nostri (Florence, 1550, 1568).[6] There may be a little local loyalty in the amount of space that Vasari gives to Piero, since Vasari

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5 For his reading of Latin we have, for instance, the fact that he seems to have read Euclid (which is, of course, no great linguistic feat), see below. For his writing, we have the testimony of marginal annotations in a copy of his perspective treatise, see Giovanna Derenzini, 'Note autografe di Piero della Francesca nel codice 616 della Bibliothèque Municipale di Bordeaux. Per la storia testuale del De prospectiva pingendi', Filologia Antica e Moderna, 9, 1995, 29 - 55.

6 Vasari made considerable revisions in some of the Lives. The most recent scholarly edition gives both versions, see G. Vasari, Le Vite dei piu eccellenti architetti, pittori e scultori italiani da Cimabue a tempi nostri, Florence, 1550, 1568, ed. P. Barocchi and R. Bettarini, Florence, 1971.

himself came from the nearby city of Arezzo.[7] Several details of what Vasari says about Piero are demonstrably incorrect — for instance his explanation of the surname 'della Francesca'[8] — but we have no good reason to doubt his assertion that Piero wrote 'many' mathematical treatises. Three of these are known to survive. The titles by which they are now known are: Trattato d'abaco, Libellus de quinque corporibus regularibus and De prospectiva pingendi. None of these texts was printed under Piero's name in the Renaissance, but all three are available in twentieth-century editions.[9] All three seem to have been originally written in

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7 This is considered in more detail in J. V. Field, Piero della Francesca: A Mathematician's Art (forthcoming).

8 This surname is now known to have been used by Piero's grandfather, also called Piero; and Vasari's explanation of it as due to the child's being brought up by a widowed mother may apply to this Piero. The painter's father died only in 1464. See James R. Banker, 'The Altarpiece of the Confraternity of the Misericordia in Borgo Sansepolcro', in M.A. Lavin (ed.), Piero della Francesca and His Legacy , Washington, D.C.: National Gallery of Art (Studies in the History of Art, no 48, Center for Advanced Study in the Visual Arts, Symposium Papers XXVIII), 1995, 21 - 35.

9 Piero della Francesca, Trattato d'abaco: Dal Codice Ashburnhamiano 280 (359*,291*) della Biblioteca Medicea Laurenziana di Firenze, ed. G. Arrighi, Pisa, 1970; Piero della Francesca, 'L'Opera "De corporibus regularibus" di Pietro dei Franceschi detto della Francesca, usurpata da Fra' Luca Pacioli', ed. G. Mancini, Memorie della R. Accademia dei Lincei, series 5, 14.8B, 1916, 441 - 580; Piero della Francesca, De prospectiva pingendi, ed. G. Nicco Fasola, Florence, 1942 (reprint Florence, 1984). All three texts are to appear in a Complete Works of Piero, under the general editorship of Marisa Dalai Emiliani, Cecil Grayson and Carlo Maccagni, of which only one volume has so far been

the vernacular, and all three are written in the style of the textbooks used in abacus schools, in which instruction proceeds through a series of worked examples, with at most only a very little linking text. However, all three works do also show connections with the learned tradition of mathematics. Here, since we are considering painting, our concern will chiefly be with the perspective treatise, but it will be useful to begin with a brief examination of the other two works.

We do not know the dates of composition of any of the three treatises. The only hints are provided by the fact that some of the problems of the Trattato appear in a neater or more developed form in the Libellus (which therefore seems to have been written later), and the fact that the dedicatory letter of the Libellus, addressed to Guidobaldo da Montefeltro (1472 - 1508) as Duke of Urbino (and therefore dating from no earlier than 1482), mentions that the Ducal library contains a copy of the perspective treatise. Unfortunately, a moment's thought shows that, in regard to dating, this latter piece of information only adds up to the perspective treatise having been written before a date after 1482. We cannot, of course, be sure whether the dedicatory letter itself is contemporary with the text or whether it was written specially for that particular copy. This unsatisfactory state of affairs in regard to the dating of Piero's writings in fact exactly mirrors the situation in regard to dating his paintings. Careful work in archives is gradually allowing us to build up a more detailed picture of his financial, civic and artistic activities, but has not so far shed any light on his writings.[10]

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published (1995), containing the Libellus, and including a facsimile of the Vatican manuscript.

10 Documents known up to 1992 are printed in the second volume of E. Battisti, Piero della Francesca, 2 vol., second edition, ed. M. Dalai Emiliani, Milan, 1992 (first edition Milan, 1971). Archival work is currently being done by James R. Banker.

The Laurentian Library in Florence has an autograph manuscript of Piero's Trattato d'abaco, whose first page bears a coat of arms that appears to be that of the Pichi family.[11] Like Piero's own family, the Pichi were associated with the Confraternity of the Misericordia in Borgo San Sepolcro, and this local connection is in accord with what Piero says in his brief introduction to the work, which begins

> Being requested that I should write some things about the abacus necessary to merchants (mercanti), by a person whose requests are to me as commands, not out of presumption but in obedience I shall steel myself, with God's help, to partly satisfy that wish, ...[12]

This is to say that the Trattato was not written for use in a school but represents a sort of ideal textbook, as may be judged by its proposed contents, which are given as Piero immediately goes on to tell us how he is to satisfy the wish

> that is by writing some examples relating to trade such as bartering (baracti), prices (merriti) and [dividing among] companions (compagnie); beginning with the rule of three things, going on to [rules of false] position and, if it please God, some things of algebra; first saying some things about fractions, ...

Many abacus books begin by telling the pupil, addressed as 'tu' and largely in the imperative, how to read Arabic numerals. This is, for example, how the Treviso Arithmetic of 1478 begins.[13] So by starting at once with fractions Piero is

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[11] Codex Ashburnham 280 (359*.291*). The first page is reproduced as the frontispiece to the edition by Arrighi (full reference in note 9 above).

[12] Florence MS 3 recto (incipit of Piero's text), Piero ed. Arrighi (full reference in note 9 above), p.39.

[13] See Frank J. Swetz, Capitalism and Arithmetic: The New Math of the 15th Century, including the full text of the Treviso Arithmetic of 1478 translated by David Eugene Smith, La Salle (Ill.): Open Court, 1987 (reprinted 1989).

assuming that he is not addressing complete beginners. Moreover, although his scheme of contents, as given in the introductory paragraph, is entirely conventional, the actual text goes far beyond what Piero had promised. 'Some things of algebra' eventually become completely abstract, with problems (some of them far from simple) posed in terms of pure numbers. There is also some geometry. In fact, there is a great deal of geometry. In the autograph manuscript in Florence, the Trattato (a small octavo volume) has 127 leaves, of which 80 verso to 120 recto are taken up with geometrical problems. It is easy to see which they are, because it is only in this part of the work that Piero provides diagrams. By twentieth-century standards, some of the arithmetical and algebraic problems should also have had diagrams, since the calculations concerned refer to parts of a geometrical figure. Despite the apparent overlap in subject matter, it would seem that Piero himself is making a rather strict division between what he considers two different kinds of problems.

Like the algebra section which precedes it, the geometry section starts with simple and conventional problems but moves on to much more abstract ones, which are of interest in their own right rather than likely to prove useful in any more practical way. For instance, we have problems not only on triangles and squares but also on the regular pentagon. Moreover, Piero gives references to relevant propositions in Euclid's Elements. Such references are sometimes found in actual school textbooks, but Piero's are far more numerous than is usual.[14] The degree of originality of Piero's advanced algebraic examples, and the methods of

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[14] For Piero's use of Euclid see Menso Folkerts, 'Piero della Francesca and Euclid', in Marisa Dalai Emiliani and Valter Curzi (eds), Piero della Francesca tra arte e scienza. (Atti del convegno internazionale di studi, Arezzo, 8 - 11 ottobre 1992, Sansepolcro, 12 ottobre 1992), Venice: Marsilio, 1996, 293 - 312.

solution employed in them, are open to dispute.[15] There are, however, no real grounds to doubt the originality of the most advanced parts of Piero's geometry. Most abacus treatises contain very little geometry. The standard practical problems are to do with finding areas of triangles (from which one could, of course, go on to find any area, by dividing it into triangles). This kind of problem truly was practical: since taxes were assessed on the basis of the area under cultivation, the landowner could be expected to take an acute interest in the work of the surveyor. Piero's father dealt in the vegetable dyestuff indigo and grew dyer's rocket (isatis tinctoria), the herb from which it is produced, so Piero must surely have had direct experience of real-life calculation of areas.

After dealing with areas of triangles, and, as already mentioned, some problems on higher polygons, Piero turns to solids. Natural philosophers of the day would certainly have heard of the five convex regular polyhedra because they are mentioned in Plato's Timæus, a text that was known throughout the Middle Ages. However, if they only knew the dialogue through Calcidius' commentary on it, they would have know little beyond the names of polyhedra other than the cube. Calcidius (or Chalcidius, fourth century AD) is not quite on the Platonic wavelength concerning the importance of mathematics. He makes very little of Plato's discussion of the properties of the elements in terms of polyhedra and the basic triangles from which their faces are constructed. Piero's discussion of the polyhedra simply refers to Euclid. It is, however, entirely in the form of numerical examples. For instance, in the first problem relating to solid bodies, the tetrahedron is introduced as follows

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15 A discussion of the algebraic content of the Trattato is given in Enrico Giusti, 'L'algebra nel Trattato d'abaco di Piero della Francesca: osservazioni e congetture', Bolletino di Storia delle Scienze Matematiche, 11.2, 1991, 55 - 83; see also R. Franci and L. Toti Rigatelli, 'Towards a History of Algebra from Leonardo of Pisa to Luca Pacioli', Janus, 72, 1985, 17 - 82.

> There is a four-faced triangular equilateral solid, and each of its sides is root 12. I ask what will be its height.[16]

The curious name for the solid — my translation reproduces Piero's turn of phrase — is presumably an indication that the correct Euclidean name would not have been of much significance. It does, in fact, signify only that the solid has four faces, so Piero has effectively translated it into the vernacular. If the reader is so inexperienced in this kind of geometry as to be in doubt of what the solid would actually look like, he can refer to a pair of diagrams at the bottom of the page. This non-explanatory style, which is normal in the abacus tradition, may, of course, be justified by the fact that the interested, or ignorant, reader could find further details elsewhere. However, following convention, Piero uses exactly the same approach in introducing a new solid

> There is a spherical body whose diameter is 6; I want to put in it a body with 8 faces, 4 triangular and 4 hexagons. I ask what its edge is.[17]

Again, a pair of diagrams (for this problem and the next one) is supplied at the bottom of the page. As far as is known, this solid, now known as a truncated tetrahedron, was new to mathematicians of Piero's time. It had been discovered by Archimedes (c.287 - 212 BC), but his work is known only through a very brief summary by Pappus of Alexandria (fl. 300 - 350 AD) in his Mathematical Collection. Pappus' text was apparently unknown to Piero, who seems to have

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[16] Florence MS 105 recto, Piero ed. Arrighi (full reference in note 9 above), p.224.

> Egl' è un quatro-base triangulare equilatero, e ciascuno suo lato è radici de 12. Domando quanto sirà il suo assis.

[17] Florence MS 107 recto, Piero ed. Arrighi (full reference in note 9 above), p.230.

> Egl' è uno corpo sperico che il suo diametro è 6; vogloci mectere dentro uno corpo de 8 base, 4 triangulare e 4 exagone. Domando del suo lato.

made an independent rediscovery of at least six of the Archimedean solids.[18] Another of them is described in the Trattato, namely the solid now known as a cuboctahedron, whose faces are six squares and eight triangles. In fact, although Piero presents both of his new solids in the same way, by means of a numerical example and a drawing, he does also say, in the course of presenting his solutions, that the truncated tetrahedron is obtained by cutting off the corners of a regular tetrahedron in such as way as to get equal sides by cutting at points one third of the way along the edge (see Fig 1a), and that the cuboctahedron is obtained by cutting corners off a cube so that cuts are made to the centre of each edge (see Fig 1b). In the Libellus de quinque corporibus regularibus there are similar accounts, that is problems and drawings, that describe the four Archimedean polyhedra produced by applying to the remaining four regular polyhedra the type of truncation that Piero applied to the tetrahedron in the Trattato (see Fig 1a), that is the truncated cube, the truncated octahedron, the truncated dodecahedron and the truncated icosahedron. There is, however, no mention of the form of truncation, to the mid-points of edges, that Piero used to get the cuboctahedron, and the cuboctahedron itself does not appear in the Libellus. This suggests that Piero distinguished between the two forms of truncation, that is, that he has not only discovered the truncated solids but has also discovered, or invented, the notion of

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18 See J. V. Field, 'Rediscovering the Archimedean polyhedra: Piero della Francesca, Luca Pacioli, Leonardo da Vinci, Albrecht Dürer, Daniele Barbaro, and Johannes Kepler', Archive for History of Exact Sciences (in press). The modern names for the individual Archimedean solids are taken from those given them by Johannes Kepler (1571 - 1630) in Harmonices mundi libri V, Linz, 1619, Book 2, Proposition 28. English translation Johannes Kepler. Five Books of the Harmony of the World, trans. and with introduction and notes by E.J. Aiton, A.M. Duncan and J.V. Field, in Transactions of the American Philosophical Society no 209 (in press since 1990).

truncation in its modern mathematical sense. However, there is no explicit mention of any such general mathematical idea in Piero's presentation of his work. In the Libellus de quinque corporibus regularibus, as in the Trattato d'abaco, we merely have worked numerical examples and drawings. The text of the Libellus is known only in one Latin manuscript (Codex urbinas 632, in the Vatican Library) and its dedicatory letter, which emphasises the importance of the patron

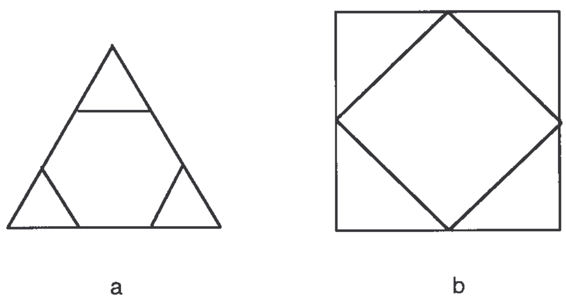

Fig 1 Truncation of faces of regular polyhedra, to obtain faces of Archimedean polyhedra.

1a Truncation to one third of edge of triangle, used by Piero in his Trattato to obtain the truncated tetrahedron. By the general form of this truncation, a regular n-gon becomes a regular 2n-gon.

1b Truncation to mid point of edge of square, used by Piero in his Trattato to obtain the cuboctahedron. By this form of truncation, a regular n-gon becomes a smaller regular n-gon.

in art and learning, is clearly designed to remind the reader of the opening paragraph of the third book of Vitruvius' De architectura. Indeed, the beginning of the letter is best described as a paraphrase of Vitruvius. However, despite these obvious links with the world of the scholar, it seems highly likely that the treatise itself was originally written in the vernacular, and its style is the same as that of the Trattato d'abaco. Perhaps conveniently, addressing the reader as 'tu' in Latin does not carry the same social message as did the 'tu' in the vernacular, so no great adjustment has to be made for the fact that in this case the putative reader is a Duke. As in the Trattato, what we have is a series of worked examples, with very little linking text. Thus both works very clearly belong to the abacus tradition. On the whole, abacus books contain a more or less standard set of examples. Piero's three-dimensional geometry is thus highly unexpected. However, it is perhaps not so very unexpected that an artist should have made these discoveries. Piero's paintings show a strong sense of three-dimensional relationships, and exactly the same sort of power of visualisation must have been required to see with the mind's eye what shape one would obtain by cutting the corners off a cube in a particular way. Nor does it seem merely accidental that the final tidying up, showing that there are exactly thirteen Archimedean solids (defined as bodies whose faces are regular polygons arranged in the same way round every vertex), should have been the work of Johannes Kepler.[19] As his work on the orbit of Mars showed, Kepler was very good indeed at thinking in three dimensions.

On perspective for painting

Piero della Francesca's treatise on perspective was written in the vernacular, but its text is also known in Latin. All texts have the Latin title De prospectiva pingendi. Piero himself seems to have co-operated in the making of the translation, which contains several improvements (in content as well as in style).[20] Most of the treatise follows the abacus book pattern in consisting of

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[19] See previous note.

[20] This is discussed more fully in J. V. Field, 'Piero della Francesca and the "distance point method" of perspective construction', Nuncius, 10.2, 1995, 509 - 530.

worked examples, solutions being presented step by step as instructions addressed to the reader ('tu'). Since the 'solution' to the problem is a drawing, the examples are not given in numerical form, though every now and then a numerical version follows, apparently provided by way of additional clarification. This method of instruction through series of worked examples was familiar not only in the abacus school but also in the painter's workshop, where an apprentice was expected to learn to draw by copying a series of drawings by his master. Workshops seem to have built up collections of such drawings, in the form of manuals. Traces of their contents can be found in repeated elements in paintings that come from the same workshop. Piero's perspective treatise is effectively a workshop manual designed to teach the apprentice to draw a series of objects in perspective. Thus although, as far as is known, it is the first treatise on perspective, it has clear antecedents within two traditions with which Piero was familiar.

However, in the first part of the treatise Piero departs from the abacus book style by presenting a series of proofs of results in geometrical optics that will be needed in the remainder of the work. The exposition is like that of Euclid's work on optics, which Piero may have known at first hand, since it was apparently widely available in Latin translation (usually under the title <u>De aspectuum varietate</u>). In any case, Piero gives references to Euclid's propositions.[21] There are also a few theorems that are purely geometrical. One of these is Book 1, section 8, in which Piero proves, by repeated use of similar triangles, that if we have a line divided into a number of parts, and the points of division are joined to a point (not on the line) and we then draw another line, parallel to the first, then the pattern of division made on the second line will be the same as that on the first one. Piero supplies a diagram like that in Fig 2a, in which the divisions along the line are equal and the point A is directly above the mid point of the line, but his

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21 See Folkerts (full reference in note 14 above).

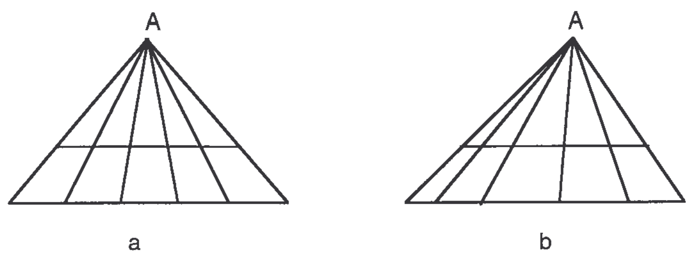

Fig 2 Diagrams for Piero della Francesca, De prospectiva pingendi, Book 1, section 8:

2a copy of figure supplied in manuscripts,

2b general figure (in which Piero's proof is still valid).

proof is entirely general, so the diagram might just as well have looked as in Fig 2b. This result may look innocent and simple enough, particularly in the symmetrical version of the diagram that is supplied in the manuscripts, but when Piero comes to use it in his discussion of perspective it in fact turns out to be of some importance. Strictly, what Piero uses is not the theorem itself, but its converse. Since the proof is by theorems relating to similar triangles, theorems that all have converses, Piero has in fact also proved the converse of this theorem (though he does not say so). Taking theoretical results for granted is, of course, characteristic of the abacus kind of mathematics, so in context Piero's silence is not sinister. On the other hand, the result he has proved is decidedly important, because when he uses it the lines converging to A have become the images of orthogonals (that is, lines which in reality were perpendicular to the picture plane). The convergence of images of orthogonals to a point 'opposite the eye', that is the foot of the perpendicular from the eye to the picture, is one of the results Alberti assumed without proof in his account of the construction of the perspective image of a chequerboard pavement in De pictura (1435). Piero realised that the result stands in need of proof. This in fact raises a problem.

Piero has proved more than his diagrams and his examples show him as using. It is possible that his proof of the theorem employed similar triangles merely in imitation of Euclid's habitual avoidance of appeals to symmetry. However, it is also possible that Piero realised the significance of avoiding such appeals, namely that one did not lose generality. Unfortunately the abacus book style of writing means that Piero does not owe us an explanation of his thinking. The matter is particularly interesting in view of the fact that if the theorem takes the form shown in Fig 2b then the use Piero makes of it, in Book 1 section 14, amounts to the recognition that the perspective images of all sets of parallel lines are sets of lines converging to points on the horizon. This result is first proved explicitly by Guidobaldo del Monte (1545 -1607) in his Perspectivæ libri sex (Pesaro, 1600).[22]

Piero's preliminary series of theorems take us as far as Book 1, section 10. Section 11 in effect deals with choices of viewing distances. After that, perspective construction begins to appear explicitly. Section 13 is notable for providing the earliest known proof of the mathematical correctness of a perspective construction. The construction is almost the same as that given, without proof, by Alberti in 1435. Since Alberti breaks his procedure down differently from Piero, the similarity is most visible only after the diagrams have been built up in a way that does violence to the line of thought of each author. For our present purposes, the most important differences between the methods are first that Piero's involves drawing fewer lines that go beyond the edge of the picture, and is therefore more practical (as will be explained later), and second

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22 For a more detailed discussion of Piero's theorem, see J. V. Field, 'When is a proof not a proof ? Some reflections on Piero della Francesca and Guidobaldo del Monte', in Proceedings of perspective conf. Rome 1995, eds M. Dalai Emiliani, F. Deuchler & R. Sinisgalli, in press.

that Piero actually proves the correctness of his construction.[23] This latter difference is no doubt not simply the result of Piero's being a much better mathematician than Alberti (which is certainly the case) but also reflects the different readerships to which the works are addressed. In De pictura, Alberti is writing about the whole of painting, dealing with the choice of subjects, the use of colours and so on, in a way that seems to be designed to interest patrons or prospective patrons. The book's being written in Latin marks it as written by a scholar and addressed to his peers. It is not at all clear who were the supposed readers of the vernacular version, Della pittura, that appeared the following year (1436). With Piero's treatise the situation is reversed. Piero is writing as a practitioner, for fellow practitioners, giving detailed instructions on how to carry out each stage of the constructions. Moreover, as he points out in the introduction to the first book of the treatise, he is dealing only with one part of painting, namely perspective. This kind of text is naturally written in the vernacular. It is not obvious who were the prospective readers of the Latin version that appeared in Piero's lifetime.[24]

After Piero has proved that his perspective construction is valid, he goes on to apply it to a series of examples. The sections immediately following the proof deal with orthogonals (section 14) and then (section 15) with the images of

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23 Pace certain historians of art, Piero's proof of his construction is perfectly correct and rigorous, though uncomfortably concise for the modern reader. See J. V. Field, 'Piero della Francesca as practical mathematician: the painter as teacher', in Marisa Dalai Emiliani and Valter Curzi (eds), Piero della Francesca tra arte e scienza, (Atti del convegno internazionale di studi, Arezzo, 8 - 11 ottobre 1992, Sansepolcro, 12 ottobre 1992), Venice: Marsilio, 1996, 331 - 354.

24 The mathematics and readerships of De pictura and De prospectiva pingendi are discussed in J. V. Field, 'Alberti, the Abacus, and Piero della Francesca's proof of perspective', Renaissance Studies, in press.

transversals, that is lines parallel to the picture plane, that are required to complete the perspective drawing of a square-tiled pavement. It is presumably for the painterly reason of avoiding having a central vertical that Piero makes his pavement 5 x 5. We are then instructed to draw the 'degraded', that is perspective, images of various other polygons in the plane. First, the corners are cut off the degraded square in such a way as to turn it into a degraded octagon (section 16), next the octagon is truncated to make a regular 16-gon (section 17). We then turn to an equilateral triangle (section 18), constructed vertex by vertex. As higher polygons follow, hexagon then pentagon, arranged so that no side is parallel to the ground line (the line of intersection of the picture plane with the ground plane), these too are constructed vertex by vertex. The drawing instructions are of mind-numbing monotony. However, they are, of course, meant not to be read but to be followed, and the repetition no doubt helped the apprentice to acquire the appropriate skill. There is no construction given for a vertex as such; each step in the construction is applied to each vertex in turn. Thus, as in an abacus book, the general rule is a matter of inference, and may indeed be of interest only to the historian rather than to the readers the original author had in mind. After the pentagon, we return to modifications of a square, which seem designed to help one construct pavements with more complicated tilings. Then there is an octagon in a general orientation (section 26), followed by a series of problems that involve drawing more than one polygon, eventually working up to the double outline required to draw the foundations of an octagonal building. Since actual octagonal buildings are not very common — in paintings they are usually identified as inspired by the Baptistery of Florence or that of San Giovanni Laterano (Rome) — Piero's problems on the octagon may partly be a homage to Brunelleschi's first perspective picture, which showed the Baptistery in Florence. On the other hand, it seems that the construction for turning a square into a regular octagon was a well known procedure, so the octagon has a fairly prominent career in works on practical mathematics. In any case, it is clear that some of the problems Piero has proposed here are not ones a painter is likely to encounter in practice, but merely exercises intended to help an apprentice acquire a basic skill. Moreover, the mathematical introduction is of no relevance except as establishing the intellectual respectability of what follows. Some insight into the thinking behind this is provided by the final section of Book 1, which is not a problem but a theorem, though it is stated rather as if it were a problem in the non-technical sense:

> To remove the error made by some who are not very experienced in this science (scienza), who say that often when they divide the degraded surface into units (bracci), the foreshortened one comes out longer than the one that has not been foreshortened; ...[25]

Luckily, the English term 'foreshortened' is like the Italian 'scurto' in implying that the line becomes shorter, though in mathematical terms it is clear that the paradox is purely verbal. There is no reason why the perspective construction should not make a line come out longer. However, it is equally clear that at least some of Piero's contemporaries, and Piero himself, regard such an outcome as unacceptable. So Piero is interested in finding out under what conditions this outcome occurs and how they may be avoided. He goes on

> ... and this happens by not understanding the distance there should be from the eye to the limit (termine) where the things are put [i.e. the picture plane], nor how wide the eye can spread the angle of its rays; so they [sc. the inexperienced] suspect perspective is not a true science (vera scientia), judging falsely because of ignorance.

There is, of course, an answer to this:

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25 Piero della Francesca, De prospectiva pingendi, Book 1, section 30, Parma, Biblioteca Palatina MS 1576 (vernacular), p. 16 verso, British Library Add. MS 10366 (Latin), p. 18 recto, Piero ed. Nicco Fasola (full reference in note 9 above) p.96.

> It is accordingly necessary to demonstrate the true distance and how far the angle at the eye can increase, so as to end their doubts.

There follows a cascade of drawing instructions that result in a diagram like that shown in Fig 3, in which the point A represents the position of the eye. The proof is then disconcertingly short. Piero subscribes to the standard belief that the eye sees ('spreads its rays') over a visual field of exactly a right angle. He thus sets out to prove that all will be well, that is that under real conditions the foreshortened length will come out shorter. In mathematical terms he needs to prove that the outcome cannot be unacceptable if the angle at A is not more than a right angle. Unfortunately, what he seems to have proved is that the outcome is unacceptable if the angle at A is more than a right angle, whereas what he needs is that the outcome should be unacceptable <u>if and only if</u> the angle at A is more than a right angle. Moreover he has neglected the effect of different heights of the eye,

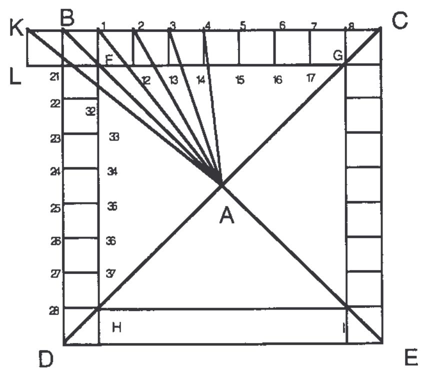

Fig 3 Figure obtained by following the drawing instructions given in Piero della Francesca, <u>De prospectiva pingendi</u>, Book 1, section 30. The point A represents the position of the eye.

which is in fact shown in the diagram as being in the ground plane.[26] However, the prescription Piero actually recommends, that the angle the picture subtends at the eye should never exceed 60°, will in fact ensure, for all the eye heights used in Piero's examples, that no foreshortened length comes out longer than its original 'perfect' length. Anyone who has properly absorbed the abacus book attitude to triangles will instantly realise that this means the viewing distance of a picture must not be less than $\sqrt{3}/2$ of the width of the picture.[27]

Since Piero's actual prescription will work as he claims, it seems possible that he knew that the proof he had presented was defective. If the result were actually correct, it would, of course, legitimate the apparently quite common practice of using a viewing distance of one half the picture width. Piero's result is, in any case, only slightly defective, in the sense that if one were to check it by drawing diagrams, the result would probably seem to be correct, since it is mainly correct to better than one part in ten. In my experience, even real mathematicians draw a few cases in the heuristic stage of an investigation, and one may surely expect an artist, or an apprentice one, to try making drawings if the mathematics became difficult. Piero seems to be starting out from the expectation that there would be an exact match between the condition imposed by perspectiva proper, that is the science of sight, and that imposed by the construction technique, which he apparently sees as an extension of the established science. The detailed mathematical preliminaries have clearly been designed to establish this connection, and, as we have seen, Piero does indeed claim that perspective for painting is a 'true science'. Piero was obviously a very competent mathematician,

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26 This may merely be a standard drawing convention, whose significance was usually nil but happens not to be nil in the present case.

27 There is a more detailed discussion of Book 1, section 30 in J. V. Field, 'Piero della Francesca's treatment of edge distortion', Journal of the Warburg and Courtauld Institutes, 49, 1986, 66 - 99 and plate 21c.

so it seems probable that he realised his proof in Book 1, section 30, was defective; but he may have believed that the result itself was true, and might have decided that this proof was the best that could be provided for this readership. With hindsight, such a calculation appears to have been justified. No-one seems to have questioned Piero's proof, and it was accepted as correct by Daniele Barbaro (1513 - 1570), who included a slightly abbreviated form of it in his widely read La Prattica della Perspettiva (Venice, 1568, 1569).[28]

The second book of De prospectiva pingendi is mainly concerned with prisms, that is, the plane figures drawn in the first book are now used as bases, and right prisms are erected on them. The examples also include combinations of prisms, for instance several hexagonal prisms are superimposed in order to obtain a hexagonal wellhead surrounded by steps (Book 2, section 6). The prisms are also sometimes modified in various ways, for example, a cube has pieces of moulding added, to make a shape like that of a pagan altar (section 7), and a larger cube becomes a house, with openings for windows and a door (section 9). The book ends, however, with another theorem, this time concerning a row of columns, the problem being to show that the ones nearer the edge of the picture do not come out wider than those nearer the middle (Book 2, section 12). In principle, this theorem concerns plane figures, since the columns are represented by their ground plans. It was presumably included in Book 2 because of its connection with architecture. Piero's proof does not seem to be quite correct, but the theorem appears to be true (as far as I can judge) and the defect of Piero's proof is only in

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28 For Barbaro's use of Piero's perspective treatise, see the paper referred to in the previous note, and M. J. Kemp, 'Piero and the Idiots: The Early Fortuna of his Theories of Perspective', in M. A. Lavin (ed.), Piero della Francesca and His Legacy, Washington, D.C.: National Gallery of Art (Studies in the History of Art, no 48, Center for Advanced Study in the Visual Arts, Symposium Papers XXVIII), 1995, 199 - 211.

his omission of some inconvenient sines and cosines. It is, however, probably an anachronism to state the matter in these terms: sines and other trigonometrical functions were well known to astronomers of the time, but seem not to have been used in abacus mathematics. (Some of the problems in Piero's Trattato could have been solved much more neatly by the use of the Sine Rule.)

Book 3 of Piero's perspective treatise deals with what he calls 'more difficult' bodies, for which he uses a different method of construction. Essentially, this method amounts to point by point projection, using a series of drawings — front view, side view and sections — to represent the original shape of the body concerned. The bodies he deals with include a mazocchio, which Piero calls a 'torculo' (section 4), a cube balanced on one vertex and with no edge parallel to the picture (section 5), the moulded base of a column (section 6), column capitals of two different designs (section 7) and a human head (section 8). The cube appears to be included just for fun, but the other examples seem practical enough, and some of them are in fact close to parts of surviving pictures by Piero. The treatise ends with instructions for making two trick drawings, one showing a goblet (rinfrescatoio) that appears to stand up from the table on which it is painted (section 10), and the other showing a ring (the kind used to suspend a lamp) that appears to hang down from the vault on which it is painted (section 11). Vasari tells us that Piero did in fact paint a goblet that seemed to stand up from the table. Both of the trick drawings are quite easy compared with the problems before them, but to make the trick effective one would, of course, also need to use painting skills that are not discussed in the perspective treatise.

Piero no doubt expected at least some of his examples to be useful in practice. He seems to have been right about this, since selections from the problems of Books 1 and 2 are to be found in many subsequent treatises (though usually in simplified form). In fact, although De prospectiva pingendi was not printed in the Renaissance, it seems to have circulated quite widely in manuscript. For instance, it was known to Egnazio Danti (1536 - 1586), who refers to it in complimentary terms in the preface to his edition of Vignola's perspective treatise Le Due regole della prospettiva prattica (Rome, 1583).[29] Much of the content of Piero's treatise became well known through Daniele Barbaro's quotation of it, sometimes verbatim and in extenso, in his Prattica della Perspettiva. However, neither Barbaro nor any other writer on perspective shows an interest in the ray-tracing method of Piero's third book. Barbaro uses the method of the second book, that is erecting prisms, to tackle problems rather more elaborate than those proposed by Piero, for instance to draw a regular dodecahedron. Barbaro in fact seems inclined to demonstrate, tacitly, that the method of Piero's third book is not really necessary. More elementary treatises tend to suggest the use of sighting instruments. These work from the real object and not, as Piero did, from a series of drawings.[30]

The fact that later generations of artists did not find the method of Piero's third book practical does not, of course, mean that he himself intended it to be purely theoretical. Indeed, some of the instructions he gives suggest very strongly that he had experience of using the method. For instance, when lines of sight are to be found, he recommends the use of a length of horsehair, preferably from the tail of a horse. Presumably these hairs are longer and straighter than those in the horse's mane? The point A represents the eye and the ruler to which Piero refers is one of those used to record the horizontal and vertical coordinates of the position in which the ray represented by the thread cuts the picture plane. What Piero says is

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29 The passage in question is discussed in J. V. Field, 'Giovanni Battista Benedetti on the Mathematics of Linear Perspective', Journal of the Warburg and Courtauld Institutes, 48, 1985, 71 - 99.

30 On the sighting instruments, see M. J. Kemp, The Science of Art: Optical themes in western art from Brunelleschi to Seurat, New Haven and London: Yale University Press, 1990.

> Now, to demonstrate the method which I intend to follow, I shall give two or three demonstrations for plane surfaces, so that through them we can arrive more easily at finding out the degradation of bodies. ... In the point .A. is fixed the nail, or if you want a needle with a very fine silk thread, it would be good to have a hair from the tail of a horse, particularly where it has to rest against the ruler; ...[31]

We may note also that Piero's choosing to start with a very simple example is an indication of a didactic purpose. It seems perfectly possible that his pupils were actually trained to carry out this procedure. At least two of Piero's pupils, Luca Signorelli (c.1441 - 1523) and Melozzo da Forlì (1438 - 1495), were notably skilful draughtsmen in later life. Carrying out this kind of procedure undoubtedly requires a firm grasp of the three-dimensional structure of the object concerned, or one is liable to lose one's way. We do, of course, know even from his mathematics that Piero was unusually good at thinking in three dimensions.

The more complicated cases in Book 3, such as that of the human head, are extremely tedious. The drawing instructions cover page after folio page of the manuscripts, and seem to be ready-made for the electronic computers that will be invented about five hundred years later. However, when we look at Piero's own practice as a painter, we can find evidence that he did actually use the procedures he describes in such overwhelming detail.[32]

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31 Piero della Francesca, De prospectiva pingendi, Book 3, section 1, Parma MS (full reference in note 25 above) p. 32 verso, BL MS (full reference in note 25 above) p. 37 verso, Piero ed. Nicco Fasola (full reference in note 9 above) p.130.

32 See J. V. Field, 'A mathematician's art', in M.A. Lavin (ed.), Piero della Francesca and His Legacy, Washington, D.C.: National Gallery of Art (Studies in the History of Art, no 48, Center for Advanced Study in the Visual Arts, Symposium Papers XXVIII), 1995, 177 - 197; and the work referred to in note 7 above.

Piero's practice as a painter

The third book of De prospectiva pingendi is preceded by an introduction. This introduction is much longer than that provided for the first book. Moreover, whereas the introduction to the first book merely distinguished what part of painting was to be the subject of the treatise, and proceeded to describe something of the author's proposed account of it, the introduction to the third book sets about defending the use of perspective. Perhaps since writing the first two books Piero had had someone point out that he had not supplied the customary 'praise' of his subject in the introduction; or perhaps Piero decided to discuss perspective here because he was aware that the examples of the third book went beyond the usual perspective practice of his time. In any case, the tone is initially defensive:

> Many painters disapprove of perspective (biasimano la prospectiva), because they do not understand the force (forza) of the lines and angles which are obtained from it; with which [lines and angles] every outline (contorno) and delineation (lineamento) is drawn in correct proportion. ...[33]

It is clear, first, that Piero is identifying perspective for painting — also sometimes called 'artificial perspective' — with perspective proper, that is the established science of sight. Second, he is asserting, apparently against the opinions of some of his contemporaries, that the geometrical reasoning of that science has the force of truth. As applied to perspectiva proper, this opinion was probably not controversial among natural philosophers of the time; painters who simply disapprove of perspective as such are indeed, as Piero implies, merely ill

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33 Piero della Francesca, De prospectiva pingendi, Book 3, introduction, Parma MS (full reference in note 25 above) p. 32 recto, BL MS (full reference in note 25 above) p. 37 recto, Piero ed. Nicco Fasola (full reference in note 9 above) p.128.

educated. However, if we take it out of its immediate context, Piero's comment on the force of lines can be made to look very like Galileo's championing of the mathematical methods of Archimedes in the study of the motion of heavy bodies. There is no reason to suppose Galileo had read Piero, or that he would have been much impressed by his work if he had read it, but it is interesting that, as in his borrowing from the theorised craft of music, Galileo is again reasoning in a way that resembles the craftsman's extension of mathematical theory into everyday practice. In the case of perspective, Piero clearly regards that extension as uncontentious. It may indeed have been so in an intellectual sense, once it was agreed, as it was by all humanist advocates of the revival of the ancient style of art, that the business of the artist was to imitate nature.[34] Piero accordingly proceeds to explain, in some detail, the relevance of what we should now call geometrical optics to the making of pictures. After this, he gives a characteristically humanist historical defence of perspective, which includes a list of the ancient artists who won eternal fame (perpetua laude) through their practice of perspective. The names are apparently mainly derived from Vitruvius' preface to the third book of De architectura.[35]

It is clear from the long introduction to the third book of De prospectiva pingendi that Piero considered a proper command of the methods of achieving optically correct perspective in pictures to be an important component of the painter's skill. At least, that is what he says. What, then, of his practice as a painter ? The writings make it clear that Piero took perspective seriously. The paintings show that he did not take it all entirely literally. Probably he would have distinguished between trick paintings intended to deceive the eye, such as the goblet and the suspension ring, and serious works of art intended to raise the mind to God or, in the case of his one surviving secular picture (a fresco of

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34 This is, for instance, made entirely explicit throughout Alberti, De pictura.

35 The list is discussed in more detail in the paper referred to in note 24 above.

Hercules, from a private house, now in the Isabella Stewart Gardner Museum, Boston, Mass.), to stir thoughts of heroic grandeur. In fact, under the normal conditions for fresco painting, where the pictures were placed at various heights on the walls of a church, there was no possibility of meeting the conditions of trompe l'œil naturalism unless all scenes were to be represented as seen from below. In a small chapel this is extremely impractical since the lines of sight would necessarily be steep. One does not need to have shared a fifteenth-century experience such as having stood too close to the wagon of a pageant, to realise the awkwardness of reading a scene from this kind of position. Pictures showing scenes from below do indeed have a special name, being said to be shown 'dal sotto in sù', but they are extremely rare. The most famous are Andrea Mantegna's decorations for the church of the Eremitani in Padua, which were unfortunately badly damaged in World War II, but are well recorded in photographs. Mantegna (1431 - 1506) was known in his day as an expert on deceiving the eye, for instance in the simulated architecture in his impressive, and delightful, decoration in the so-called 'painted room' ('camera picta') in the Ducal palace at Mantua,[36] but one suspects that general opinion in the fifteenth century was in agreement with that in the twentieth that the steep dal sotto in sù perspective in the Eremitani frescos made the scenes difficult to read. Mantegna was a hard act to follow in any circumstances, but in this case his example must have seemed more like a warning (at least to artists). The artist would probably not have been entirely free to take his own decisions on the matter, but the patron might have drawn similar

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36 This is also sometimes known as the 'camera degli sposi'. It shows various members of the Gonzaga family, but is chiefly famous for the painted opening in the dome, through which spectators appear to look down on the scene in the room. Mantegna makes extremely skilful use of the natural lighting in achieving his effects.

lessons: commissions seem often to have been based on emulating some successful scheme elsewhere.

On the technical level, Piero was certainly capable of designing pictures that took account of the low viewpoint of the normal spectator (that is a spectator who did not have the benefit of scaffolding). As we shall see, he did just this in his fresco of the Resurrection, in what was then the Council chamber of the town hall of Borgo San Sepolcro (and is now a room in the Museo Civico). However, we shall begin by looking at the fresco cycle The Story of the True Cross (in the chancel of the church of San Francesco, Arezzo), which Piero designed and painted in the period from about 1451 until, at latest, 1466. This is not very precise dating, but the fact that several examples in Book 3 of De prospectiva pingendi bear a considerable resemblance to parts of the pictures in San Francesco suggests that their origin may have been in practical reality, and that the diagrams accompanying them may be copies after real working drawings. There is thus rather good evidence that Piero was using what he knew about perspective in designing the pictures in this fresco cycle. So some mathematics almost certainly went into their overall design as well as into the details that we can compare directly with what we find in the treatise. However, it is a quite different matter to get the mathematics out again, that is to reconstruct the mathematical structure of the pictures. It is only for a very few of them that we even have a large enough number of more or less secure orthogonals to allow us to find what Alberti calls the 'centric point', the point which is the foot of the perpendicular from the eye of the ideal observer to the picture. In no case do we have any secure means of finding the ideal viewing distance. Thus at best we can find eye heights (for the ideal observer), though we may also note that on the left wall the 'centric points' seem to be aligned on the central vertical. This last fact is not at all surprising but is probably worth noting, since it gives us a little help in examining the perspective of the right wall, where only the middle tier (showing scenes of the Queen of Sheba) has any convenient orthogonals. Such analysis as one can perform shows that Piero is consistent, over the four sets of pictures on the two side walls and the two parts of the altar wall, in arranging that the eye level of the ideal observer is below that of standing characters in the scenes in question, so that one does have a sensation of looking upwards. However, since Piero's point A, the 'centric point', is the point to which the eye of the ideal observer is directed, Piero regards it as obvious (but nevertheless reminds us in his treatise)[37] that this point must lie within the picture itself. It is accordingly impossible for the eye height to be close to the actual eye height of an observer standing on the floor of the church. Piero has in fact made the height of each tier a little greater than that of the one below, to allow for the apparent diminution due to increasing distance from the eye. The eye heights in this fresco cycle run entirely contrary to the prescription recorded by Alberti in De pictura, where the height of the centric point is explicitly set at the eye height of a standing figure in the picture. The Albertian rule (if one may call it that) is in fact followed in Masaccio's famous fresco The Tribute Money (Brancacci Chapel, Santa Maria del Carmine, Florence), painted in the mid 1420s, which Piero certainly knew and must surely have greatly admired. Piero's concern to set an ideal eye height that is at least qualitatively correct even if far from precisely accurate, is also found in the various scenes in the altarpiece he painted for the convent of Sant'Antonio in Perugia, which seems to date from 1468 - 69. (The altarpiece is now in the Galleria Nazionale dell' Umbria, Perugia.) In the lowest scenes, those of the narrative predella, the eye height of the ideal observer is actually higher than that of standing figures in the pictures, which suggests that the ideal viewer was the officiant.

Only the officiant would have been close enough to see much of the exquisite detail that has been included in the pictures of the Sant'Antonio altarpiece, so it is

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37 The reminder comes in the last few lines of Book 1, section 13, following Piero's proof that his perspective construction is correct.

perhaps not surprising to find he has been privileged in another way as well. However, the details of the Arezzo frescos are also, in many passages, far too small to be seen by a normal observer, on the floor of the church, and in this case the only privileged observer would have to be on scaffolding. This quality of finish seems to be the norm in Piero's works, whether in fresco, tempera or oil paint. It shows a carefulness which is very much at variance with romantic notions of artistic creation and helps to remind us that to Piero's contemporaries, a painter was a craftsman. One may see this concern with very precise rendering of detail as connected with Piero' well-attested interest in Netherlandish art[38] or, on another level, as an indication that he was not exclusively interested in the effect upon the viewer but worked also 'ad majorem Dei gloriam'. Making things perfect for the eye of God is, of course, entirely compatible with using all one's scientific understanding of the laws of light to do so. Whether for God or for the spectator, Piero seems to have interpreted his task as being to get things to look right by following the dictates of scientific understanding. However, there cannot be any question of the pictures becoming predominantly scientific (in any meaningful sense of the word) since Piero's art, and his sensibility as an artist, belong within a long and vigorous pictorial tradition in which representation is a means and not an end in itself. To put it in less abstract terms: Piero knows the difference between a painting and a theorem. The fact that he also visibly gives some importance to theorems is one of the things that makes his work interesting.

There is, of course, no question as to which tendency will win in the case of a clash between the demands of optical accuracy and the demands of producing a satisfactory work of art. Piero's paintings show us the painter as clearly as his

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38 For a recent account, see Bert W. Meijer, 'Piero and the North', in M. A. Lavin (ed.), Piero della Francesca and His Legacy, Washington, D.C.: National Gallery of Art (Studies in the History of Art, no 48, Center for Advanced Study in the Visual Arts, Symposium Papers XXVIII), 1995, 143 - 159.

writings show us the mathematician. The Resurrection provides a particularly clear example. The picture is painted in fresco on the end wall of what was the Council Chamber in the Town Hall of Borgo San Sepolcro. The choice of subject is an allusion to the fact that the town takes its name from the Holy Sepulchre, and the position of the picture, rather high on the wall, was no doubt determined by the position and design of the seating for the councillors. The scene, which is almost exactly square, is framed by simulated architecture, in the form of an opening flanked by columns. This architectural framing provides a few very short orthogonals, which meet on the central vertical axis of the picture, well below the eye height of the sleeping soldiers who lie sprawled in the foreground. The figures of the soldiers are also drawn in a way that suggests we are looking up at them, though it is not possible to work out what the ideal eye height may be. Behind the soldiers, Christ is rising from the tomb, and we can see the upper surface of His left foot, which is placed on the edge of the sarcophagus. This view implies an eye level well above that given by the orthogonals and implied by the figures of the soldiers. Moreover, the remainder of the figure of Christ gives the impression that we are seeing it straight on. It is obvious that the picture has not been designed so that the perspective is exactly correct for one particular viewing position, on the floor or on scaffolding. However, the picture is not confusing. In fact, it is extremely imposing and dignified. The analysis just given is not part of a normal response to the fresco, and making it does not (in my case, or that of anyone I know) in any way change how one actually responds to the picture. In a certain sense the analysis is to do with 'reading' the picture in purely technical terms, and has little to do with how one sees it. However, Piero himself must have thought things through. Frescos have to be planned carefully, and while one is painting one cannot see the total effect.

It is, of course, possible to produce various theological and practical rationalisations of what Piero has done. One could say, for instance, that the soldiers belong to our world whereas Christ exists in a space that is not a mere extension of theirs. The difference in viewpoint thus emphasises the miraculous nature of what is being shown. On the practical side, one may note that, since the room is a large one, this picture is going to be seen from a wide variety of positions, so it might be a good idea to avoid a rigid perspective scheme in which deviation from the correct viewing position might become noticeable. We have no way of knowing what Piero's reasoning was, but it is certain that it resulted in an extremely effective picture, greatly admired by Piero's contemporaries and by Vasari, which is nevertheless very far from mathematically correct in its perspective. A fairly similar use of multiple 'ideal' eye levels is found in several pictures by Mantegna, most notably in his engraving of the Resurrection, in which the two saints, one on each side of the Risen Christ, are seen from below whereas Christ himself is seen straight on. As in the Piero fresco, one notices not discomfort but rather a certain tension that is entirely in keeping with the religious significance of the scene.

Even quite simple analysis, say with transparent rulers placed on a small photograph, shows that many fifteenth-century pictures are in flagrant violation of the supposed rules of naturalistic perspective. The tolerance of the eye in reading pictures as showing something three-dimensional even when the drawing is not exactly correct is, of course, well known to today's perceptual psychologists. Fifteenth-century artists must at least have known that they could get away with 'errors' in their mathematics. The Resurrection fresco, and a number of other pictures, show that Piero knew this. However, in one surviving picture by Piero, The Flagellation of Christ (panel, 59 x 81.5 cm, Galleria Nazionale delle Marche, Urbino), the perspective does indeed seem to be exactly correct. In the absence of other suitable examples, this picture appears in countless histories of art, unfortunately usually without an acknowledgement that its correctness makes it at least an extreme rarity. As the picture is small, there is no question of trompe l'œil, but detailed measurements have amply confirmed that the perspective is indeed correct.[39] The ideal eye height is well below that of standing figures, and the viewing distance is about two and a half times the picture width. The only deviations from mathematical correctness are in the width of part of the paving (where the small adjustment has an obvious explanation in terms of the composition as a whole) and in part of the architecture at one side (where a compositional explanation can be constructed). In fact, the mathematical structure of the picture is so visible, for instance in the complicated paving and the array of square coffers deployed across the ceiling of the room we are looking into, that one may be forgiven for wondering whether Piero was responding to a challenge, or at least to a commission to display his mathematical skill to the full. Here, as in the Arezzo frescos, but this time in miniature, we find many elements that occur as examples in Piero's perspective treatise. In front of the original painting, one is distracted by the brilliance of the colour and the wonderfully subtle rendering of textures and of light, but when faced with a black and white reproduction of the picture it is possible to envisage it as having been conceived as a pictorial accompaniment to the treatise. There is, however, one inconsistency: given the exquisite detail of the paint handling, Piero surely cannot have expected a viewer to remain at the ideal distance (of about two metres). In fact, the modelling is so strong that even from far too close one sees things as solid.

In the matter of the correctness of their perspective, most of Piero's pictures are closer to the Arezzo fresco cycle than to either the Resurrection or the Flagellation. That is to say that, where one can make measurements, the results suggest Piero has taken care over the perspective, which conforms with the standard of the examples in his treatise. All Piero's pictures convey a strong sense of three-dimensional structure. It accordingly seems likely that even where one

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39 See M.A. Lavin, Piero della Francesca. The Flagellation of Christ, New York, 1972 (reprint, with additional bibliography, Chicago, 1990).

cannot make precise measurements, and must, for instance, fall back on comparing sizes of human figures or trees (as in the Baptism, now in the National Gallery, London), there may have been some calculation, which is now irrecoverable, but is visible in the sense that the pictures look entirely convincing.[40]

What makes a science?

In Piero's time, a painter was considered to be a craftsman, since he used his hands to make a living. It is clear that in De pictura Alberti is engaged upon an enterprise that, by associating the new style of painting with the humanist revival of ancient learning, tends to raise the status of the artist. Alberti indeed cites from Pliny some names of artists who were of high social rank.[41] Moreover, notwithstanding his omission of any explanation of the connection between the 'cone of vision' and the proposed perspective construction, and his rather cursory treatment of actual pieces of mathematics, Alberti lays considerable stress on the fact that in order to paint well one must have a good understanding of perspectiva, including its mathematical part. In context this seems partly to be a claim that painters, who use such learned material in their work, should be accorded a standing more like that associated with the pursuit of the learned mathematical sciences. However, as Alberti undoubtedly knew, perspectiva was not one of the four accepted mathematical sciences. It was merely a 'mixed' science, called a science because it could indeed partly be studied mathematically, but also including much that was studied in a qualitative way by the methods of natural

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40 A much fuller discussion of this aspect of Piero's work will be found in the book referred to in note 7 above.

41 Alberti, De pictura, §28, from Pliny, Naturalis historia, XXXV, 133 and elsewhere.

philosophy, such as theories of the nature of light. In contrast to Alberti's treatise, which considers the whole of painting, Piero's De prospectiva pingendi is, as we have seen, and as Piero explicitly states in his introduction, concerned only with the part of painting that has to do with 'artificial' perspective, and it is accordingly entirely mathematical, thus making clear its connection with the elements in perspectiva proper that made it a 'science'.

Piero's practice as a painter suggests, however, that he wished to go further than this. As we know from his text, he considered 'perspective for painting' to be a science. The text suggests he was, moreover, prepared to take great pains to apply that science in his art, but it does not deal even with all the elements of simple optics that we find in the paintings. Piero may have made calculations as to what should be shown in the reflective haloes worn by the saints in the Sant'Antonio altarpiece, and it seems highly likely he did make calculations about where the shadows should fall in the ceiling shown in the Flagellation. However, in many passages in his painting there are details that must be due simply to acute observation, for instance the placing of highlights in different positions on each of the seed pearls round the upper edge of the boots worn by St Michael in the panel now in the National Gallery, London. In all his works Piero is apparently striving for a completeness of representation that is very much in the spirit of the humanist interpretation of the demands of classical authorities such as Pliny. It is, also, in line with the careful combination of observation and theory that we find in the drawings of Piero's younger contemporary Leonardo da Vinci (1452 - 1519), and indeed in Leonardo's few completed works of art. The spirit of exploration in Piero, as in Leonardo, seems to me to be perfectly in accord with the precepts of the Aristotelian natural philosophy and sciences of their times. During the period of his closest friendship with Luca Pacioli (c.1445 - 1514), Leonardo became very interested in mathematics, but on the whole it is Piero who has the greater taste for abstraction. For instance, when drawing polyhedra, Leonardo shows what is clearly a portrait of a model, whereas Piero merely draws an outline, with no indication of the flow of light. Unfortunately, we have no drawings by Piero other than mathematical ones, so it is not reasonable to attempt to push the comparison too far. All the same, Piero's art does seem to be scientific by the standards of his time. It is thus perhaps no accident that his invocation of the 'force of lines' should sound so like statements Galileo made more than 150 years later about the power of mathematics.